Name ______________________________ Class ____________ Date ____________

Reteaching

Lines and Angles

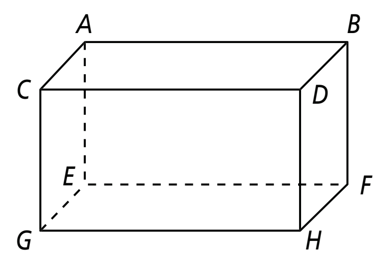
Not all lines and planes intersect.

- Planes that do not intersect are *parallel planes.*
- Lines that are in the same plane and do not intersect are *parallel.*
- The symbol $\parallel$ shows that lines or planes are parallel: $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$ means "Line *AB* is parallel to line *CD*."
- Lines that are not in the same plane and do not intersect are *skew*.

Parallel planes: plane *ABDC* $\parallel$ plane *EFHG*

plane *BFHD* $\parallel$ plane *AEGC*

plane *CDHG* $\parallel$ plane *ABFE*

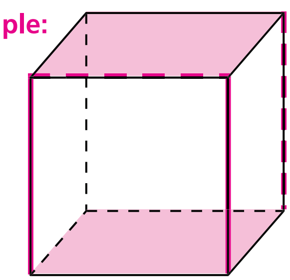


Examples of parallel lines: $\overleftrightarrow{CD} \parallel \overleftrightarrow{AB} \parallel \overleftrightarrow{EF} \parallel \overleftrightarrow{GH}$

Examples of skew lines: $\overleftrightarrow{CD}$ is skew to $\overleftrightarrow{BF}$, $\overleftrightarrow{AE}$, $\overleftrightarrow{EG}$, and $\overleftrightarrow{FH}$.

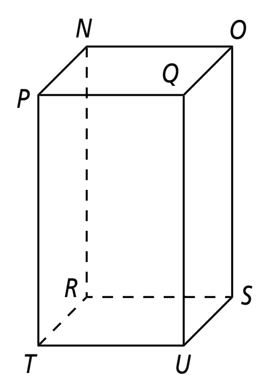
Exercises

In Exercises 1–3, use the figure at the right. **Answers may vary. Sample:**



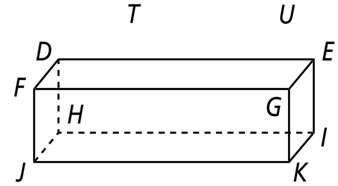
1. Shade one set of parallel planes.
2. Trace one set of parallel lines with a solid line.
3. Trace one set of skew lines with a dashed line.

In Exercises 4–7, use the diagram to name each of the following.



4. a line that is parallel to $\overleftrightarrow{RS}$
   Answers may vary. Sample: $\overleftrightarrow{NO}$
5. a line that is skew to $\overleftrightarrow{QU}$
   Answers may vary. Sample: $\overleftrightarrow{RS}$
6. a plane that is parallel to *NRTP*
   plane *OSUQ*
7. three lines that are parallel to $\overleftrightarrow{OQ}$
   $\overleftrightarrow{NP}$, $\overleftrightarrow{RT}$, and $\overleftrightarrow{SU}$

In Exercises 8–11, describe the statement as *true* or *false*. If *false*, explain.



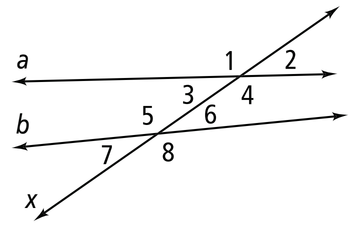
8. plane *HIKJ* $\parallel$ plane *IEGK*
   False; these planes intersect.
9. $\overleftrightarrow{DH} \parallel \overleftrightarrow{GK}$
   true
10. $\overleftrightarrow{HJ}$ and $\overleftrightarrow{HD}$ are skew lines.
    False; the lines intersect.
11. $\overleftrightarrow{FG} \parallel \overleftrightarrow{KI}$ **False; the lines are in different planes, and since they do not intersect they are skew.**

Name ______________________ Class __________ Date __________

Reteaching (continued)

Lines and Angles

The diagram shows lines *a* and *b* intersected by line *x*.



Line *x* is a transversal. A transversal is a line that intersects two or more lines found in the same plane.

The angles formed are either interior angles or exterior angles.

Interior Angles

between the lines cut by the transversal

∠3, ∠4, ∠5, and ∠6 in diagram above

Exterior Angles

outside the lines cut by the transversal

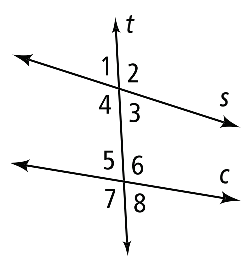
∠1, ∠2, ∠7, and ∠8 in diagram above

Four types of special angle pairs are also formed.

Angle Pair	Definition	Examples
alternate interior	inside angles on opposite sides of the transversal, not a linear pair	∠3 and ∠6 ∠4 and ∠5
alternate exterior	outside angles on opposite sides of the transversal, not a linear pair	∠1 and ∠8 ∠2 and ∠7
same-side interior	inside angles on the same side of the transversal	∠3 and ∠5 ∠4 and ∠6
corresponding	in matching positions above or below the transversal, but on the same side of the transversal	∠1 and ∠5 ∠3 and ∠7 ∠2 and ∠6 ∠4 and ∠8

Exercises

Use the diagram at the right to answer Exercises 12–15.



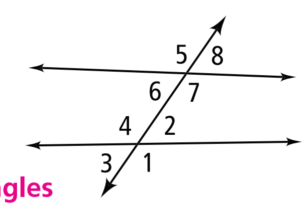
12. Name all pairs of corresponding angles.
∠1 and ∠5; ∠2 and ∠6; ∠3 and ∠8; ∠4 and ∠7

13. Name all pairs of alternate interior angles.
∠4 and ∠6; ∠3 and ∠5

14. Name all pairs of same-side interior angles.
∠4 and ∠5; ∠3 and ∠6

15. Name all pairs of alternate exterior angles.
∠1 and ∠8; ∠2 and ∠7

Use the diagram at the right to answer Exercises 16 and 17. Decide whether the angles are *alternate interior angles, same-side interior angles, corresponding,* or *alternate exterior angles.*



16. ∠1 and ∠5
alternate exterior angles

17. ∠4 and ∠6
same-side interior angles

Name ______________________________ Class ____________ Date ____________

Reteaching

Properties of Parallel Lines

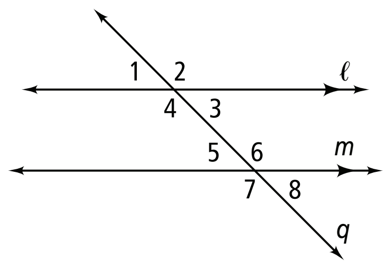
When a transversal intersects parallel lines, special supplementary and congruent angle pairs are formed.

Supplementary angles formed by a transversal intersecting parallel lines:

- same-side interior angles (Postulate 3-1)

$m\angle 4 + m\angle 5 = 180 \qquad m\angle 3 + m\angle 6 = 180$

Congruent angles formed by a transversal intersecting parallel lines:



- alternate interior angles (Theorem 3-1)

$\angle 4 \cong \angle 6 \qquad \angle 3 \cong \angle 5$

- corresponding angles (Theorem 3-2)

$\angle 1 \cong \angle 5 \qquad \angle 2 \cong \angle 6$

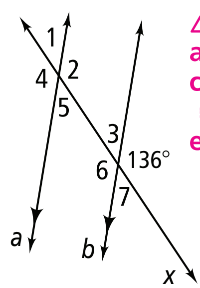
$\angle 4 \cong \angle 7 \qquad \angle 3 \cong \angle 8$

- alternate exterior angles (Theorem 3-3)

$\angle 1 \cong \angle 8 \qquad \angle 2 \cong \angle 7$

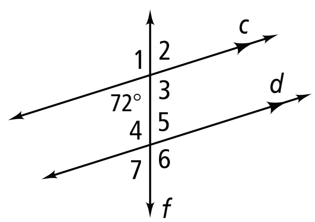
Identify all the numbered angles congruent to the given angle. Explain.

1.



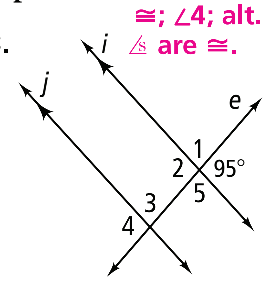
∠6; vert. ∠s are ≅; ∠2; corresp. ∠s are ≅; ∠4; alt. ext. ∠s are ≅.

2.



∠2; vert. ∠s are ≅; ∠5; alt. int. ∠s are ≅; ∠7; corresp. ∠s are ≅.

3.

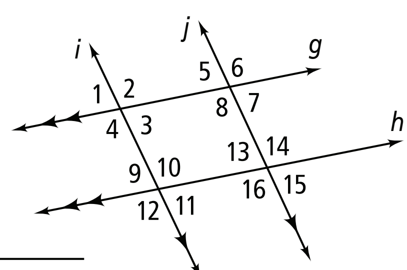


∠2; vert. ∠s are ≅; ∠4; alt. ext. ∠s are ≅.

4. Supply the missing reasons in the two-column proof.

Given: $g \parallel h$, $i \parallel j$

Prove: $\angle 1$ is supplementary to $\angle 16$.



Statements	Reasons
1) $\angle 1 \cong \angle 3$	1) __?__ **Vertical angles are congruent.**
2) $g \parallel h$; $i \parallel j$	2) Given
3) $\angle 3 \cong \angle 11$	3) __?__ **Corresponding angles are congruent.**
4) $\angle 11$ and $\angle 16$ are supplementary.	4) __?__ **Same-side interior angles are supplementary.**
5) $\angle 1$ and $\angle 16$ are supplementary.	5) __?__ **Substitution property**

Name ______________________ Class ____________ Date ____________

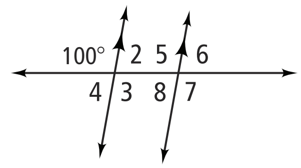
Reteaching (continued)

Properties of Parallel Lines

You can use the special angle pairs formed by parallel lines and a transversal to find missing angle measures.

Problem

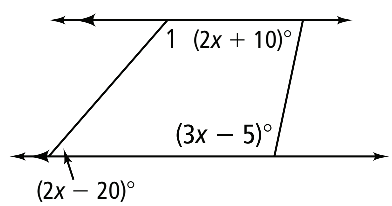
If $m\angle 1 = 100$, what are the measures of $\angle 2$ through $\angle 8$?



Supplementary angles:	$m\angle 2 = 180 - 100$	$m\angle 2 = 80$
	$m\angle 4 = 180 - 100$	$m\angle 4 = 80$
Vertical angles:	$m\angle 1 = m\angle 3$	$m\angle 3 = 100$
Alternate exterior angles:	$m\angle 1 = m\angle 7$	$m\angle 7 = 100$
Alternate interior angles:	$m\angle 3 = m\angle 5$	$m\angle 5 = 100$
Corresponding angles:	$m\angle 2 = m\angle 6$	$m\angle 6 = 80$
Same-side interior angles:	$m\angle 3 + m\angle 8 = 180$	$m\angle 8 = 80$

Problem

What are the measures of the angles in the figure?



$(2x + 10) + (3x - 5) = 180$	Same-Side Interior Angles Postulate
$5x + 5 = 180$	Combine like terms.
$5x = 175$	Subtract 5 from each side.
$x = 35$	Divide each side by 5.

Find the measure of these angles by substitution.

$2x + 10 = 2(35) + 10 = 80$ $\quad$ $3x - 5 = 3(35) - 5 = 100$

$2x - 20 = 2(35) - 20 = 50$

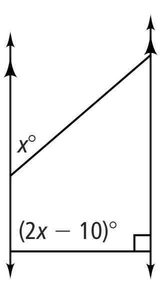
To find $m\angle 1$, use the Same-Side Interior Angles Postulate:

$50 + m\angle 1 = 180$, so $m\angle 1 = 130$

Exercises

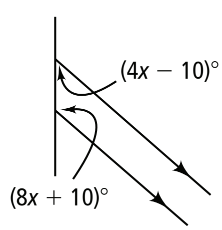
Find the value of x. Then find the measure of each labeled angle.

5.



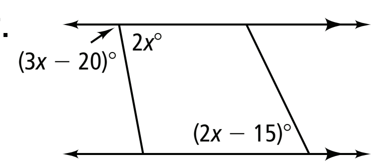
50; 50; 90

6.



15; 130; 50

7.



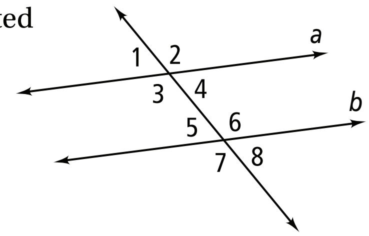
40; 100; 80; 65

Name ______________________ Class ____________ Date ____________

Reteaching

Proving Lines Parallel

Special angle pairs result when a set of parallel lines is intersected by a transversal. The converses of the theorems and postulates in Lesson 3-2 can be used to prove that lines are parallel.



Theorem 3-4: Converse of Corresponding Angles Theorem

If $\angle 1 \cong \angle 5$, then $a \parallel b$.

Theorem 3-5: Converse of the Alternate Interior Angles Theorem

If $\angle 3 \cong \angle 6$, then $a \parallel b$.

Theorem 3-6: Converse of the Same-Side Interior Angles Postulate

If $\angle 3$ is supplementary to $\angle 5$, then $a \parallel b$.

Theorem 3-7: Converse of the Alternate Exterior Angles Theorem

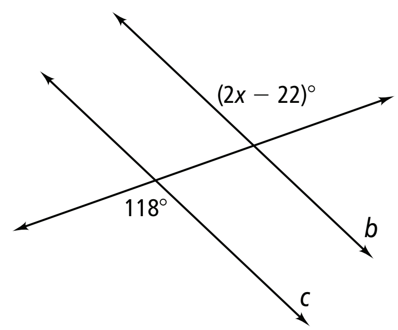
If $\angle 2 \cong \angle 7$, then $a \parallel b$.

Problem

For what value of x is $b \parallel c$?

The given angles are alternate exterior angles. If they are congruent, then $b \parallel c$.

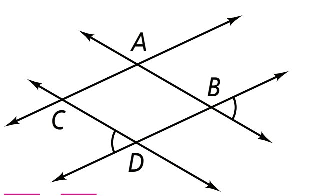
$$2x - 22 = 118$$
$$2x = 140$$
$$x = 70$$



Exercises

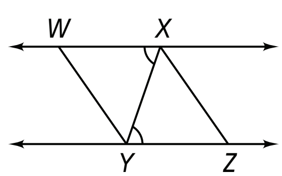
Which lines or line segments are parallel? Justify your answers.

1.



$\overline{AB} \parallel \overline{CD}$ because the $\cong$ ∡s are alt. ext. angles.

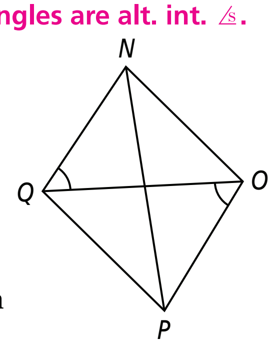
2.



$\overline{WX} \parallel \overline{YZ}$ because the $\cong$ ∡s are alt. int. ∡s.

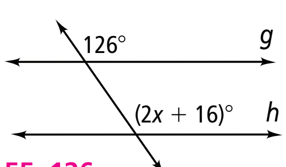
3.

$\overline{OP} \parallel \overline{QN}$ because the $\cong$ angles are alt. int. ∡s.



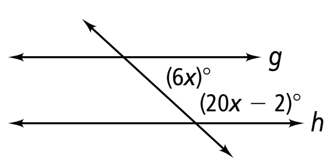
Find the value of x for which $g \parallel h$. Then find the measure of each labeled angle.

4.



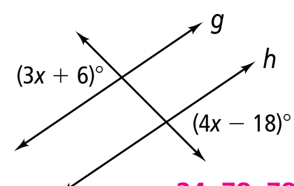
55; 126

5.



7; 42; 138

6.



24; 78; 78

Name ______________________ Class ____________ Date ____________

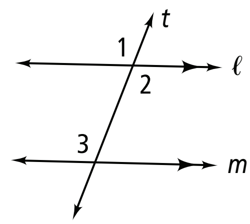
Reteaching (continued)

Proving Lines Parallel

A flow proof is a way of writing a proof and a type of graphic organizer. Statements appear in boxes with the reasons written below. Arrows show the logical connection between the statements.

Problem

Write a flow proof for Theorem 3-1: If a transversal intersects two parallel lines, then alternate interior angles are congruent.



Given: $\ell \parallel m$

Prove: $\angle 2 \cong \angle 3$

$\boxed{\ell \parallel m}$ → $\boxed{\angle 1 \cong \angle 3}$ → $\boxed{\angle 2 \cong \angle 3}$

Given — If $\parallel$ lines, then corresponding $\measuredangle$s are $\cong$.

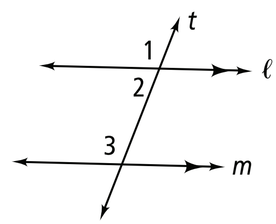
$\boxed{\angle 1 \cong \angle 2}$ → $\boxed{\angle 2 \cong \angle 3}$

Vertical angles are $\cong$.

Transitive Property of $\cong$

Exercises

Complete a flow proof for each.



7. Complete the flow proof for Theorem 3-2 using the following steps. Then write the reasons for each step.

a. $\angle 2$ and $\angle 3$ are supplementary. **b.** $\angle 1 \cong \angle 3$

c. $\ell \parallel m$ **d.** $\angle 1$ and $\angle 2$ are supplementary.

Theorem 3-2: If a transversal intersects two parallel lines, then same side interior angles are supplementary.

Given: $\ell \parallel m$

Prove: $\angle 2$ and $\angle 3$ are supplementary.

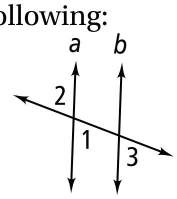
$\boxed{\ell \parallel m}$ (Given) → $\boxed{\angle 1 \cong \angle 3}$ (corr. $\measuredangle$s $\cong$) → $\boxed{\angle 2 \text{ and } \angle 3 \text{ are supplementary.}}$ (Substitution)

$\boxed{\angle 1 \text{ and } \angle 2 \text{ are supplementary.}}$ (linear pair) → $\boxed{\angle 2 \text{ and } \angle 3 \text{ are supplementary.}}$

8. Write a flow proof for the following:

Given: $\angle 2 \cong \angle 3$

Prove: $a \parallel b$



$\boxed{\angle 2 \cong \angle 3}$ (Given) → $\boxed{\angle 3 \cong \angle 1}$ (Substitution) → $\boxed{a \parallel b}$ (If $\cong$ corresponding $\measuredangle$s, then lines are parallel.)

$\boxed{\angle 2 \cong \angle 1}$ (Vertical $\measuredangle$s are $\cong$.) → $\boxed{\angle 3 \cong \angle 1}$

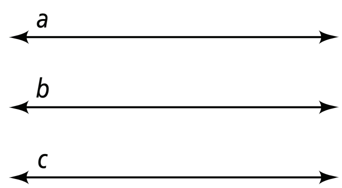
Name ______________________________ Class ____________ Date ____________

Reteaching

Parallel and Perpendicular Lines

You can use angle pairs to prove that lines are parallel. The postulates and theorems you learned are the basis for other theorems about parallel and perpendicular lines.

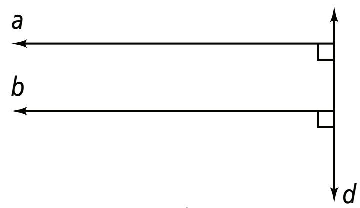
Theorem 3-8: Transitive Property of Parallel Lines



If two lines are parallel to the same line, then they are parallel to each other.

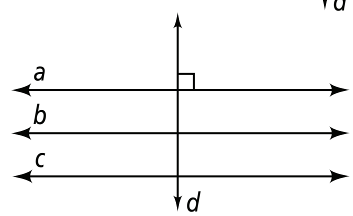
If $a \parallel b$ and $b \parallel c$, then $a \parallel c$. Lines a, b, and c can be in different planes.

Theorem 3-9: If two lines are perpendicular to the same line, then those two lines are parallel to each other.



This is only true if all the lines are in the same plane. If $a \perp d$ and $b \perp d$, then $a \parallel b$.

Theorem 3-10: Perpendicular Transversal Theorem

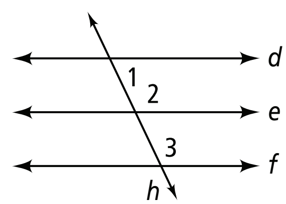


If a line is perpendicular to one of two parallel lines, then it is also perpendicular to the other line.

This is only true if all the lines are in the same plane. If $a \parallel b$ and c, and $a \perp d$, then $b \perp d$, and $c \perp d$.

Exercises

1. Complete this paragraph proof of Theorem 3-8.

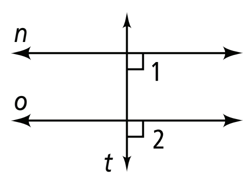


Given: $d \parallel e$, $e \parallel f$

Prove: $d \parallel f$

Proof: Because it is given that $d \parallel e$, then $\angle 1$ is supplementary to $\angle 2$ by the __Same-Side Int. Angles__ Postulate. Because it is given that $e \parallel f$, then $\angle 2 \cong \angle 3$ by the __Corresponding Angles__ Theorem. So, by substitution, $\angle 1$ is supplementary to $\angle$ __3__. By the __Converse of the Same-Side Int. Angles__ Postulate, $d \parallel f$.

2. Write a paragraph proof of Theorem 3-9.



Given: $t \perp n$, $t \perp o$

Prove: $n \parallel o$

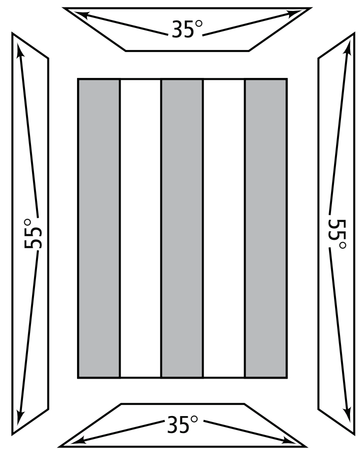
Given that $t \perp n$ and $t \perp o$, $m\angle 1 = 90$ and $m\angle 2 = 90$, by def. of perpendicular lines. Thus $\angle 1 \cong \angle 2$. So, $n \parallel o$ because of the Converse of the Corr. ⦞ Thm.

Name ______________________ Class ____________ Date ____________

Reteaching (continued)

Parallel and Perpendicular Lines

Problem



A carpenter is building a cabinet. A decorative door will be set into an outer frame.

a. If the lines on the door are perpendicular to the top of the outer frame, what must be true about the lines?

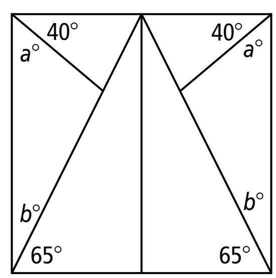
b. The outer frame is made of four separate pieces of molding. Each piece has angled corners as shown. When the pieces are fitted together, will each set of sides be parallel? Explain.

c. According to Theorem 3-9, lines that are perpendicular to the same line are parallel to each other. So, since each line is perpendicular to the top of the outer frame, all the lines are parallel.

Know	Need	Plan
The angles for the top and bottom pieces are 35°. The angles for the sides are 55°.	Determine whether each set of sides will be parallel.	Draw the pieces as fitted together to determine the measures of the new angles formed. Use this to decide if each set of sides will be parallel.

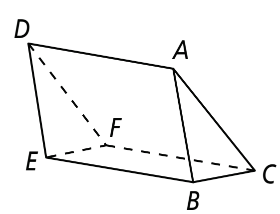
The new angle is the sum of the angles that come together. Since $35 + 55 = 90$, the pieces form right angles. Two lines that are perpendicular to the same line are parallel. So, each set of sides is parallel.

Exercises



3. An artist is building a mosaic. The mosaic consists of the repeating pattern shown at the right. What must be true of a and b to ensure that the sides of the mosaic are parallel?
$a = 50$ and $b = 25$

4. Error Analysis A student says that according to Theorem 3-10, if $\overleftrightarrow{AD} \parallel \overleftrightarrow{CF}$ and $\overleftrightarrow{AD} \perp \overleftrightarrow{AB}$, then $\overleftrightarrow{CF} \perp \overleftrightarrow{AB}$. Explain the student's error.
$\overleftrightarrow{AB}$ and $\overleftrightarrow{CF}$ are in different planes.



Name ______________________ Class __________ Date __________

Reteaching

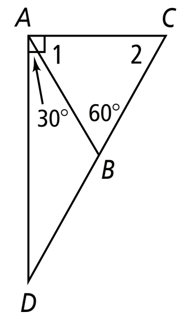
Parallel Lines and Triangles

Triangle Angle-Sum Theorem:

The measures of the angles in a triangle add up to 180.

Problem

In the diagram at the right, $\triangle ACD$ is a right triangle.
What are $m\angle 1$ and $m\angle 2$?



Step 1

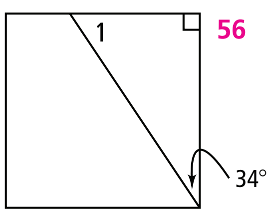
$m\angle 1 + m\angle DAB = 90$	Angle Addition Postulate
$m\angle 1 + 30 = 90$	Substitution Property
$m\angle 1 = 60$	Subtraction Property of Equality

Step 2

$m\angle 1 + m\angle 2 + m\angle ABC = 180$	Triangle Angle-Sum Theorem
$60 + m\angle 2 + 60 = 180$	Substitution Property
$m\angle 2 + 120 = 180$	Addition Property of Equality
$m\angle 2 = 60$	Subtraction Property of Equality

Exercises

Find $m\angle 1$.

1. 

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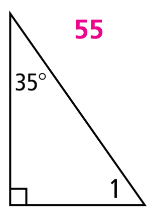
2. 

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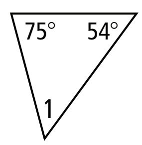
3. 

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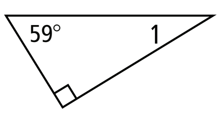
4. 

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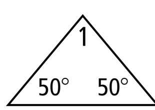
5. 

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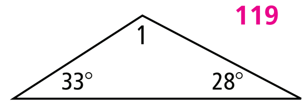
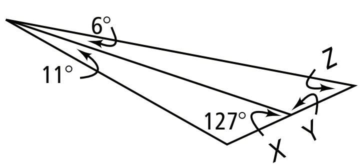
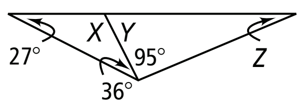
6. 

119

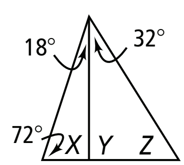
Algebra **Find the value of each variable.**

7. 

42; 138; 36

8. 

117; 63; 22

9. 

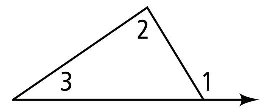
90; 90; 58

Name ______________________ Class ____________ Date ____________

Reteaching (continued)

Parallel Lines and Triangles

In the diagram at the right, $\angle 1$ is an exterior angle of the triangle. An exterior angle is an angle formed by one side of a polygon and an extension of an adjacent side.



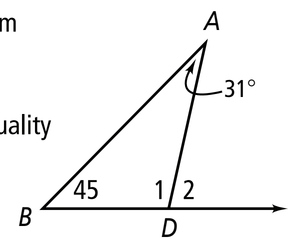
For each exterior angle of a triangle, the two interior angles that are not next to it are its remote interior angles. In the diagram, $\angle 2$ and $\angle 3$ are remote interior angles to $\angle 1$.

The *Exterior Angle Theorem* states that the measure of an exterior angle is equal to the sum of its remote interior angles. So, $m\angle 1 = m\angle 2 + m\angle 3$.

Problem

What are the measures of the unknown angles?

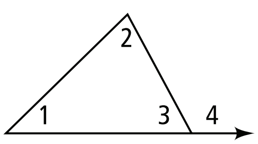
$m\angle ABD + m\angle BDA + m\angle BAD = 180$	Triangle Angle-Sum Theorem
$45 + m\angle 1 + 31 = 180$	Substitution Property
$m\angle 1 = 104$	Subtraction Property of Equality
$m\angle ABD + m\angle BAD = m\angle 2$	Exterior Angle Theorem
$45 + 31 = m\angle 2$	Substitution Property
$76 = m\angle 2$	Subtraction Property of Equality



Exercises

What are the exterior angle and the remote interior angles for each triangle?

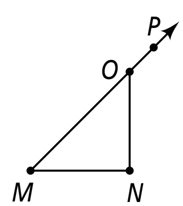
10.



exterior: **∠4**

interior: **∠1, ∠2**

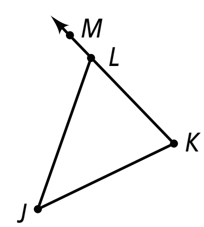
11.



exterior: **∠NOP**

interior: **∠OMN, ∠MNO**

12.

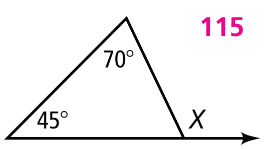


exterior: **∠JLM**

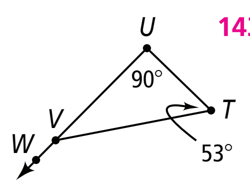
interior: **∠JKL, ∠LJK**

Find the measure of the exterior angle.

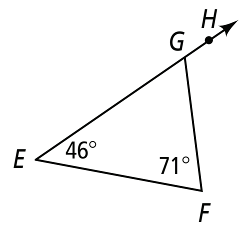
13. **115**



14. **143**



15. **117**



Name ______________________________ Class ____________ Date ____________

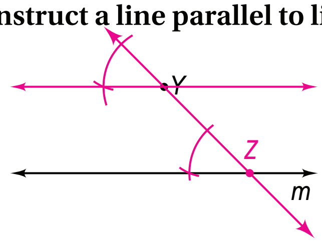
Reteaching (continued)

Constructing Parallel and Perpendicular Lines

Exercises

Construct a line parallel to line *m* and through point *Y*. Sample answers shown.

1.



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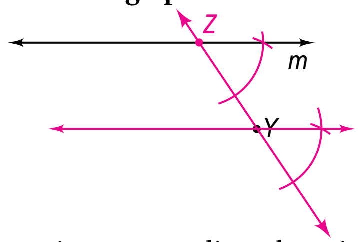

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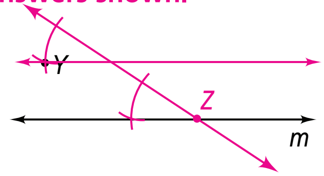
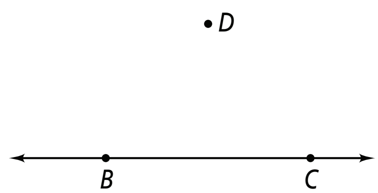


Perpendicular Postulate Through a point not on a line, there is exactly one line perpendicular to the given line.

Problem

Given: Point *D* not on $\overleftrightarrow{BC}$

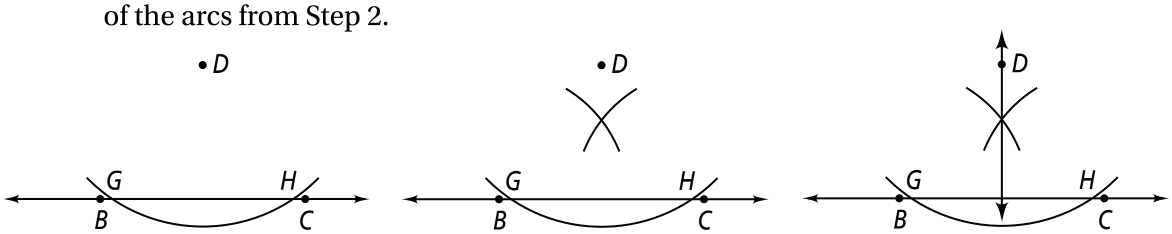


Construct: a line perpendicular to $\overleftrightarrow{BC}$ through *D*

Step 1 Construct an arc centered at *D* that intersects $\overleftrightarrow{BC}$ at two points. Label those points *G* and *H*.

Step 2 Construct two arcs of equal length centered at points *G* and *H*.

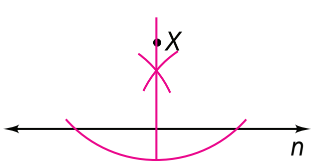
Step 3 Construct the line through point *D* and the intersection of the arcs from Step 2.



Step 1 Step 2 Step 3

Construct a line perpendicular to line *n* and through point *X*. Sample answers shown.

4.



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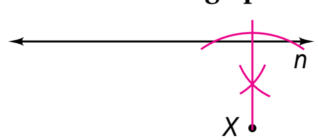

6.