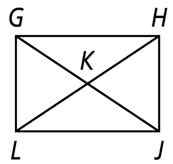
Name ______________________________ Class ____________ Date ____________

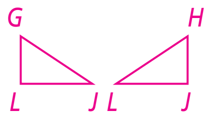
Reteaching (continued)

Congruence in Overlapping Triangles

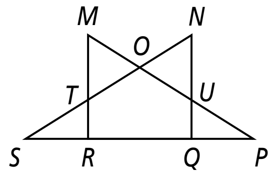
Separate and redraw the overlapping triangles. Identify the vertices.

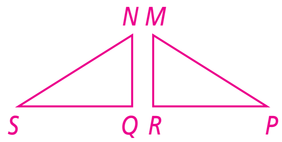
1. $\triangle GLJ$ and $\triangle HJL$



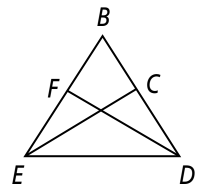


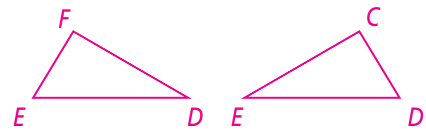
2. $\triangle MRP$ and $\triangle NQS$





3. $\triangle FED$ and $\triangle CDE$

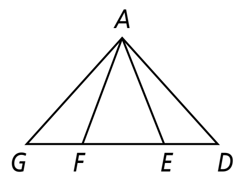




Fill in the blanks for the two-column proof.

4. Given: $\angle AEG \cong \angle AFD$, $\overline{AE} \cong \overline{AF}$, $\overline{GE} \cong \overline{FD}$

Prove: $\triangle AFG \cong \triangle AED$



Statements	Reasons
1) $\angle AEG \cong \angle AFD$, $\overline{AE} \cong \overline{AF}$, $\overline{GE} \cong \overline{FD}$	1) ? **Given**
2) ? **$\triangle AEG \cong \triangle AFD$**	2) SAS
3) $\overline{AG} \cong \overline{AD}$, $\angle G \cong \angle D$	3) ? **CPCTC**
4) ? **$\overline{GE} \cong \overline{FD}$**	4) Given
5) $GE = FD$	5) ? **Def. of $\cong$**
6) $GF + FE = GE$, $FE + ED = FD$	6) ? **Seg. Addition Post.**
7) $GF + FE = FE + ED$	7) ? **Substitution Property**
8) ? **$GF = ED$**	8) Subtr. Prop. of Equality
9) ? **$\triangle AFG \cong \triangle AED$**	9) ? **SAS**

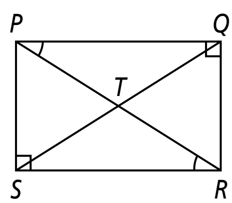
Use the plan to write a two-column proof.

5. Given: $\angle PSR$ and $\angle PQR$ are right angles, $\angle QPR \cong \angle SRP$.

Prove: $\triangle STR \cong \triangle QTP$

Plan for Proof:

Prove $\triangle QPR \cong \triangle SRP$ by AAS. Then use CPCTC and vertical angles to prove $\triangle STR \cong \triangle QTP$ by AAS.



Statements: 1) $\angle PSR$ and $\angle PQR$ are rt. $\angle$s; $\angle QPR \cong \angle SRP$; 2) $\angle PSR \cong \angle RQP$; 3) $\overline{PR} \cong \overline{RP}$; 4) $\triangle QPR \cong \triangle SRP$; 5) $\angle STR \cong \angle QTP$; 6) $\overline{PQ} \cong \overline{RS}$; 7) $\triangle STR \cong \triangle QTP$; Reasons: 1) Given; 2) Rt. $\angle$s are congruent; 3) Refl. Prop. of $\cong$; 4) AAS; 5) Vert. $\angle$s are $\cong$; 6) CPCTC; 7) AAS

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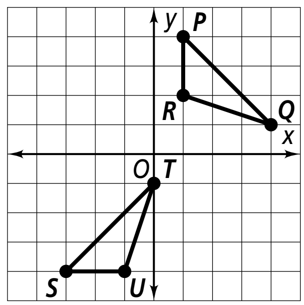
Reteaching

Congruence Transformations

Because rigid motions preserve distance and angle measure, the image of a rigid motion or composition of rigid motions is congruent to the preimage. Congruence can be defined by rigid motions as follows.

Two figures are *congruent* if and only if there is a sequence of one or more rigid motions that map one figure onto the other.

Because rigid motions map figures to congruent figures, rigid motions and compositions of rigid motions are also called *congruence transformations*. If two figures are congruent, you can find a congruence transformation that maps one figure to the other.

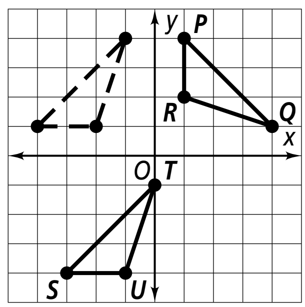


Problem

In the figure at the right, $\triangle PQR \cong \triangle STU$. What is a congruence transformation that maps $\triangle PQR$ to $\triangle STU$?

$\triangle STU$ appears to have the same shape and orientation as $\triangle PQR$, but rotated 90°, so start by applying the rotation $r_{(90°,\, O)}$ on the vertices of $\triangle PQR$.

$r_{(90°,\, O)}(P) = (-4, 1)$, $r_{(90°,\, O)}(Q) = (-1, 4)$, $r_{(90°,\, O)}(r) = (-2, 1)$



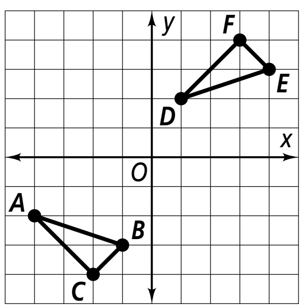
Graph the image $r_{(90°,\, O)}(\triangle PQR)$. A translation of 1 unit to the right and 5 units down maps the image to $\triangle STU$.

Therefore, $(T_{<1,\, -5>} \circ r_{(90°,\, O)})(\triangle PQR) = \triangle STU$.

Exercises

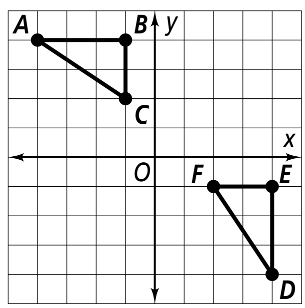
Find a congruence transformation that maps $\triangle ABC$ to $\triangle DEF$.

1.



Answers may vary. Sample:
$(R_{x\text{-axis}} \circ T_{<5,\, 0>})(\triangle ABC) = \triangle DEF$

2.



Answers may vary. Sample:
$(r_{(270°,\, O)} \circ R_{y\text{-axis}})(\triangle ABC) = \triangle DEF$

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Reteaching (continued)

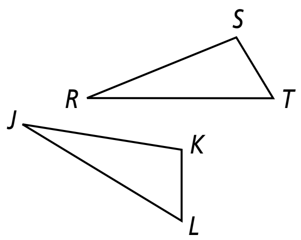
Congruence Transformations

If you can show that a congruence transformation exists from one figure to another, then you have shown that the figures are congruent.

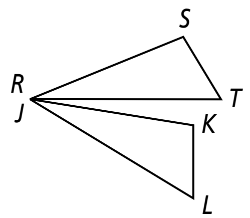
Problem

Verify the SSS Postulate by using a congruence transformation.

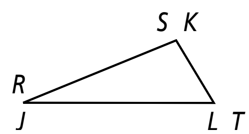
Given: $\overline{JK} \cong \overline{RS}$, $\overline{KL} \cong \overline{ST}$, $\overline{LJ} \cong \overline{TR}$
Prove: $\triangle JKL \cong \triangle RST$



Start by translating $\triangle JKL$ so that points J and R coincide.



Because you are given that $\overline{JK} \cong \overline{RS}$, there is a rigid motion that maps $\overline{JK}$ onto $\overline{RS}$ by rotating $\triangle JKL$ about point R so that $\overline{JK}$ and $\overline{RS}$ coincide. Thus, there is a congruence transformation that maps $\triangle JKL$ to $\triangle RST$, so $\triangle JKL \cong \triangle RST$.

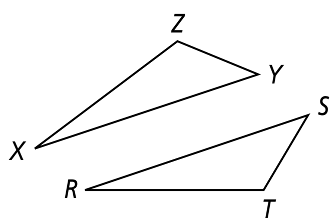


Exercises

3. Verify the SAS Postulate for triangle congruence by using congruence transformations.

Given: $\angle R \cong \angle X$, $\overline{RS} \cong \overline{XY}$, $\overline{ST} \cong \overline{YZ}$
Prove: $\triangle RST \cong \triangle XYZ$

Answers may vary. Sample: Since $\overline{RS} \cong \overline{XY}$, translate $\triangle RST$ so $\overline{RS}$ coincides with $\overline{XY}$. Then reflect $\triangle RST$ across $\overline{XY}$ to complete the transformation.



4. Verify the ASA Postulate for triangle congruence by using congruence transformations.

Given: $\angle A \cong \angle J$, $\angle B \cong \angle K$, $\overline{AB} \cong \overline{JK}$
Prove: $\triangle ABC \cong \triangle JKL$

Answers may vary. Sample: Translate $\triangle ABC$ so that points C and L coincide. Then rotate $\triangle ABC$ about point L until $\overline{AB}$ and $\overline{JK}$ coincide.