Name ______________________ Class ____________ Date ____________

Reteaching (continued)

The Polygon Angle-Sum Theorems

Exercises

Find the sum of the interior angles of each polygon.

1. quadrilateral **360** **2.** octagon **1080** **3.** 18-gon **2880**

4. decagon **1440** **5.** 12-gon **1800** **6.** 28-gon **4680**

Find the measure of an interior angle of each regular polygon. Round to the nearest tenth if necessary.

7. decagon **144** **8.** 12-gon **150** **9.** 16-gon **157.5**

10. 24-gon **165** **11.** 32-gon **168.8** **12.** 90-gon **176**

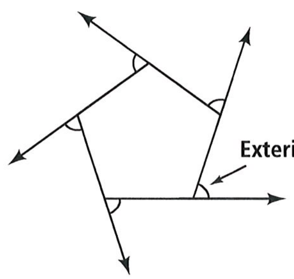
Exterior Angles of a Polygon

The exterior angles of a polygon are those formed by extending sides. There is one exterior angle at each vertex.

Polygon Exterior Angle-Sum Theorem:

The sum of the measures of the exterior angles of a polygon is 360.

A pentagon has five exterior angles. The sum of the measures of the exterior angles is always 360, so each exterior angle of a regular pentagon measures 72.



Exercises

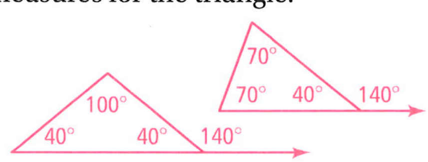
Find the measure of an exterior angle for each regular polygon. Round to the nearest tenth if necessary.

13. octagon **45** **14.** 24-gon **15** **15.** 34-gon **10.6**

16. decagon **36** **17.** heptagon **51.4** **18.** hexagon **60**

19. 30-gon **12** **20.** 28-gon **12.9** **21.** 36-gon **10**

22. **Draw a Diagram** A triangle has two congruent angles, and an exterior angle that measures 140. Find two possible sets of angle measures for the triangle. Draw a diagram for each. **40, 40, 100; 40, 70, 70**



Name ______________________ Class ____________ Date ____________

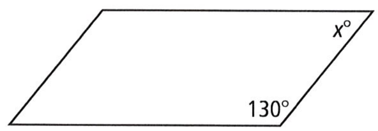
Reteaching (continued)

Properties of Parallelograms

Exercises

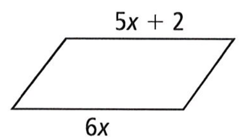
Find the value of *x* in each parallelogram.

1.



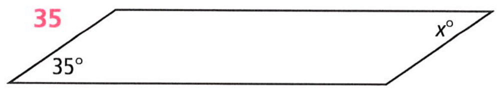
50

2.

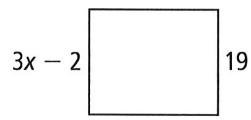


2

3. 35

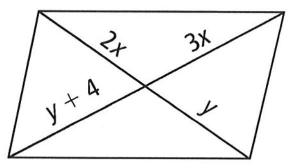


4.



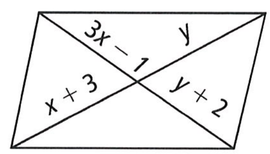
7

5.



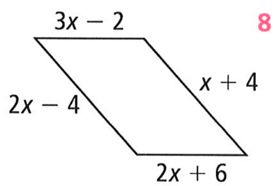
4

6.



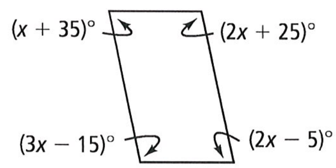
3

7.



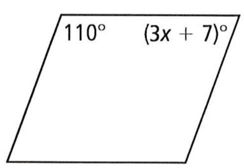
8

8.



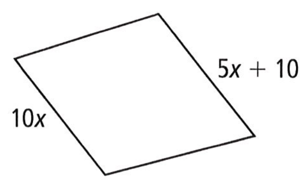
40

9.



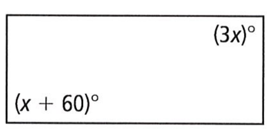
21

10.



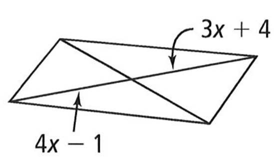
2

11.



30

12.



5

13. Writing Write a statement about the consecutive angles of a parallelogram.
Consecutive angles of a parallelogram are supplementary.

14. Writing Write a statement about the opposite angles of a parallelogram.
Opposite angles of a parallelogram are congruent.

15. Reasoning One angle of a parallelogram is 47. What are the measures of the other three angles in the parallelogram?
47, 133, and 133

Name ______________________ Class __________ Date __________

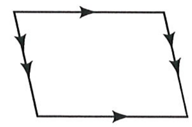
Reteaching

Proving That a Quadrilateral Is a Parallelogram

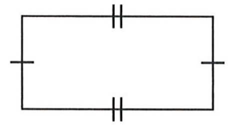
Is a quadrilateral a parallelogram?

There are five ways that you can confirm that a quadrilateral is a parallelogram.

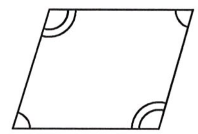
If both pairs of opposite sides are parallel, then the quadrilateral is a parallelogram.



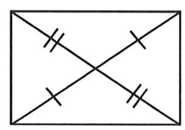
If both pairs of opposite sides are congruent, then the quadrilateral is a parallelogram.



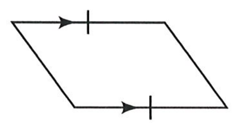
If both pairs of opposite angles are congruent, then the quadrilateral is a parallelogram.



If the diagonals bisect each other, then the quadrilateral is a parallelogram.

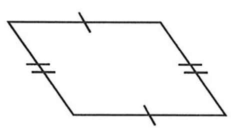


If one pair of sides is both congruent and parallel, then the quadrilateral is a parallelogram.

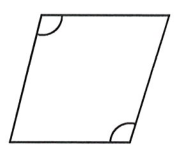


Exercises

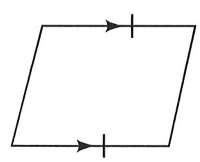
Can you prove that the quadrilateral is a parallelogram based on the given information? Explain.

1. 

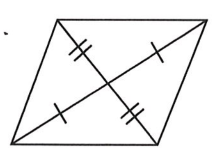
yes; opposite sides $\cong$

2. 

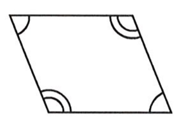
no; not enough info

3. 

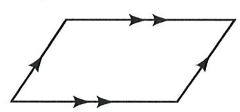
yes; 1 pair of sides $\cong$ and $\parallel$

4. 

Yes; diagonals bisect each other.

5. 

yes; opposite $\angle$s $\cong$

6. 

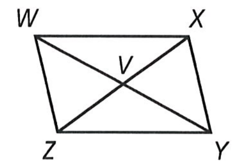
yes; opposite sides $\parallel$

Name ______________________ Class ____________ Date ____________

Reteaching (continued)

Proving That a Quadrilateral Is a Parallelogram

Determine whether the given information is sufficient to prove that quadrilateral *WXYZ* is a parallelogram.



7. $\overline{WY}$ bisects $\overline{ZX}$ no

8. $\overline{WX} \parallel \overline{ZY}$; $\overline{WZ} \cong \overline{XY}$ no

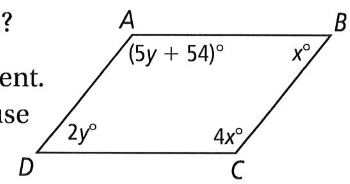
9. $\overline{VZ} \cong \overline{VX}$; $\overline{WX} \cong \overline{YZ}$ no

10. $\angle VWZ \cong \angle VYX$; $\overline{WZ} \cong \overline{XY}$ yes

You can also use the requirements for a parallelogram to solve problems.

Problem

For what value of x and y must figure *ABCD* be a parallelogram?



In a parallelogram, the two pairs of opposite angles are congruent. So, in *ABCD*, you know that $x = 2y$ and $5y + 54 = 4x$. You can use these two expressions to solve for x and y.

Step 1: Solve for y.

	$5y + 54 = 4x$	
	$5y + 54 = 4(2y)$	Substitute $2y$ for x.
	$5y + 54 = 8y$	Simplify.
	$54 = 3y$	Subtract $5y$ from each side.
	$18 = y$	Divide each side by 3.

Step 2: Solve for x.

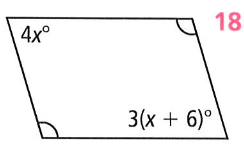
	$x = 2y$	Opposite angles of a parallelogram are congruent.
	$x = 2(18)$	Substitute 18 for y.
	$x = 36$	Simplify.

For *ABCD* to be a parallelogram, x must be 36 and y must be 18.

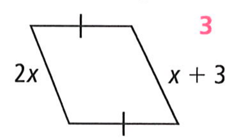
Exercises

For what value of x must the quadrilateral be a parallelogram?

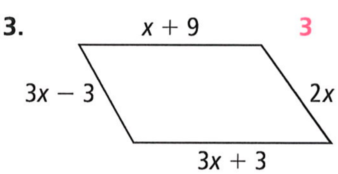
11. 18



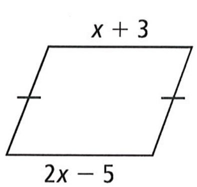
12. 3



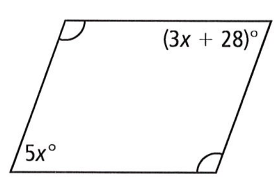
13. 3



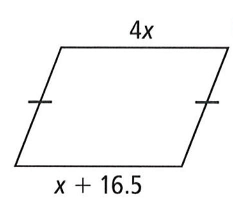
14. 8



15. 14



16. 5.5



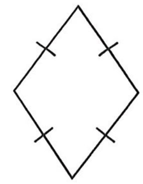
Name ______________________________ Class ____________ Date ____________

Reteaching

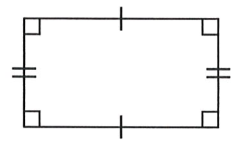
Properties of Rhombuses, Rectangles, and Squares

Rhombuses, rectangles, and *squares* share some characteristics. But they also have some unique features.

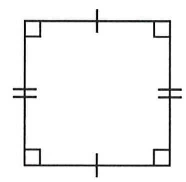
A rhombus is a parallelogram with four congruent sides.



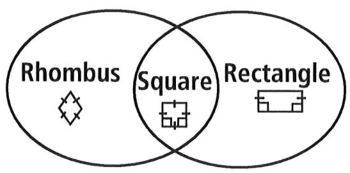
A rectangle is a parallelogram with four congruent angles. These angles are all right angles.



A square is a parallelogram with four congruent sides and four congruent angles. A square is both a rectangle and a rhombus. A square is the only type of rectangle that can also be a rhombus.



Here is a Venn diagram to help you see the relationships.



There are some special features for each type of figure.

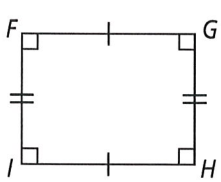
Rhombus: The diagonals are perpendicular.
The diagonals bisect a pair of opposite angles.

Rectangles: The diagonals are congruent.

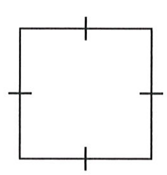
Squares: The diagonals are perpendicular.
The diagonals bisect a pair of opposite angles (forming two 45° angles at each vertex).
The diagonals are congruent.

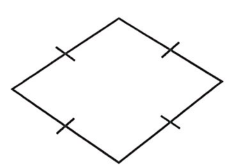
Exercises

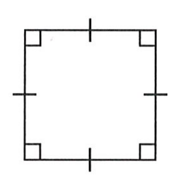
Decide whether the parallelogram is a rhombus, a rectangle, or a square.

1. 

rectangle

2. 
rhombus

3. 
rhombus

4. 
square

Name ______________________ Class ____________ Date ____________

Reteaching (continued)

Properties of Rhombuses, Rectangles, and Squares

List the quadrilaterals that have the given property. Choose among *parallelogram, rhombus, rectangle,* and *square.*

5. Opposite angles are supplementary.
rectangle, square

6. Consecutive sides are ≅.
rhombus, square

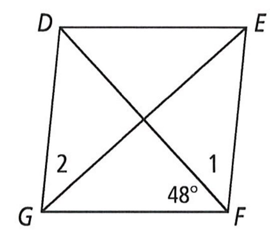
7. Consecutive sides are ⊥.
rectangle, square

8. Consecutive angles are ≅.
rectangle, square

You can use the properties of rhombuses, rectangles, and squares to solve problems.

Problem

Determine the measure of the numbered angles in rhombus *DEFG*.



$\angle 1$ is part of a bisected angle. $m\angle DFG = 48$, so $m\angle 1 = 48$.

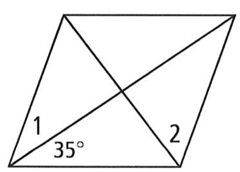
Consecutive angles of a parallelogram are supplementary.
$m\angle EFG = 48 + 48 = 96$, so $m\angle DGF = 180 - 96 = 84$.

The diagonals bisect the vertex angle, so $m\angle 2 = 84 \div 2 = 42$.

Exercises

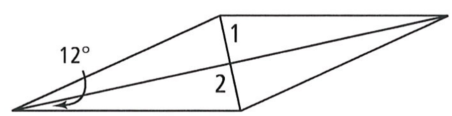
Determine the measure of the numbered angles in each rhombus.

9.



35; 55

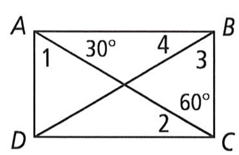
10.



78; 90

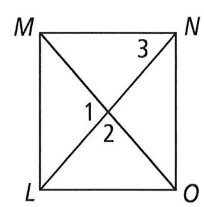
Determine the measure of the numbered angles in each figure.

11. rectangle *ABCD*



60; 30; 60; 30

12. square *LMNO*



90; 90; 45

Algebra ***TUVW* is a rectangle. Find the value of *x* and the length of each diagonal.**

13. $TV = 3x$ and $UW = 5x - 10$
5; 15; 15

14. $TV = 2x - 4$ and $UW = x + 10$
14; 24; 24

15. $TV = 6x + 4$ and $UW = 4x + 8$
2; 16; 16

16. $TV = 7x + 6$ and $UW = 9x - 18$
12; 90; 90

17. $TV = 8x - 2$ and $UW = 5x + 7$
3; 22; 22

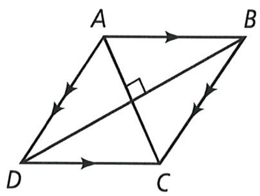
18. $TV = 10x - 4$ and $UW = 3x + 24$
4; 36; 36

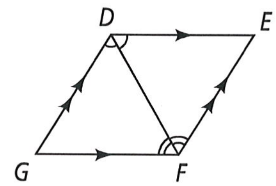
Name ______________________ Class ____________ Date ____________

Reteaching

Conditions for Rhombuses, Rectangles, and Squares

A parallelogram is a rhombus if either of these conditions is met:



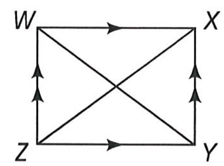


1) The diagonals of the parallelogram are perpendicular. (Theorem 53)

2) A diagonal of the parallelogram bisects a pair of opposite angles. (Theorem 54)

A parallelogram is a rectangle if the diagonals of the parallelogram are congruent.

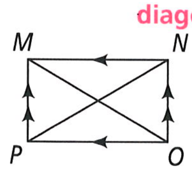
$\overline{WY} \cong \overline{XZ}$



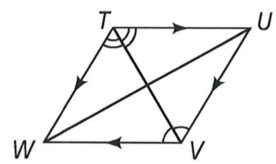
Exercises

Classify each of the following parallelograms as a *rhombus*, a *rectangle*, or a *square*. For each, explain.

1. $\overline{MO} \cong \overline{PN}$ **Rectangle; the diagonals are ≅.**

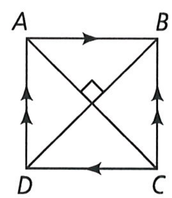


2.



Rhombus; the diagonals bisect opposite angles.

3. $\overline{AC} \cong \overline{BD}$ **Square; the diagonals are ≅ and ⊥.**



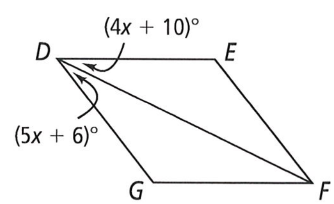
Use the properties of rhombuses and rectangles to solve problems.

Problem

For what value of x is $\square DEFG$ a rhombus?

In a rhombus, diagonals bisect opposite angles.

So, $m\angle GDF = m\angle EDF$.



$(4x + 10) = (5x + 6)$	Set angle measures equal to each other.
$10 = x + 6$	Subtract 4x from each side.
$4 = x$	Subtract 6 from each side.

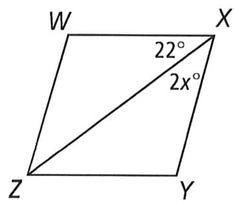
Name ______________________________ Class ____________ Date ____________

Reteaching (continued)

Conditions for Rhombuses, Rectangles, and Squares

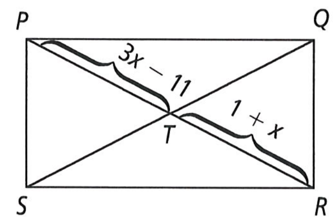
Exercises

4. For what value of x is ▱*WXYZ* a rhombus? 11



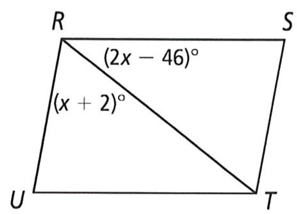
5. $SQ = 14$. For what value of x is ▱*PQRS* a rectangle?

Solve for *PT*. Solve for *PR*. 6; 7; 14

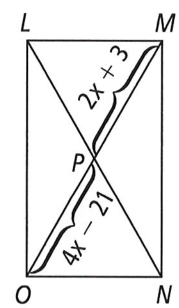


6. For what value of x is ▱*RSTU* a rhombus? What is $m\angle SRT$? What is $m\angle URS$?

48; 50; 100



7. $LN = 54$. For what value of x is ▱*LMNO* a rectangle? 12



8. Given: ▱*ABCD*, $\overline{AC} \perp \overline{BD}$ at *E*.

Prove: *ABCD* is a rhombus.

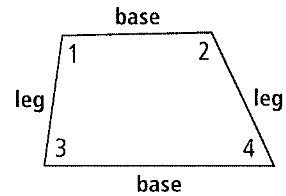
Statements	Reasons
1) $\overline{AE} \cong \overline{CE}$	1) ? Diagonals of a ▱ bisect each other.
2) $\overline{AC} \perp \overline{BD}$ at E	2) ? Given
3) ? $\angle AED$ and $\angle CED$ are right angles.	3) Definition of perpendicular lines
4) ? $\angle AED \cong \angle CED$	4) ? All right angles are congruent.
5) ? $\overline{DE} \cong \overline{DE}$	5) Reflexive Property of Congruence
6) $\triangle AED \cong \triangle CED$	6) ? SAS Postulate
7) $\overline{AD} \cong \overline{CD}$	7) ? CPCTC
8) ? $\overline{AB} \cong \overline{CD}$, $\overline{AD} \cong \overline{BC}$	8) Opposite sides of a ▱ are ≅.
9) ? $\overline{AB} \cong \overline{BC} \cong \overline{CD} \cong \overline{DA}$	9) ? Transitive Property of Congruence
10) *ABCD* is a rhombus.	10) ? Definition of rhombus

Name ______________________ Class ____________ Date ____________

Reteaching

Trapezoids and Kites

A *trapezoid* is a quadrilateral with exactly one pair of parallel sides. The two parallel sides are called *bases.* The two nonparallel sides are called *legs.*



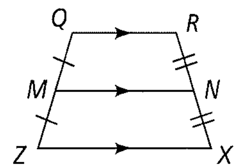
A pair of base angles share a common base.

$\angle 1$ and $\angle 2$ are one pair of base angles.

$\angle 3$ and $\angle 4$ are a second pair of base angles.

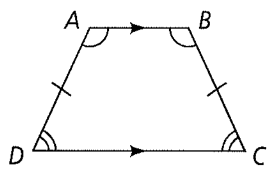
In any trapezoid, the *midsegment* is parallel to the bases. The length of the midsegment is half the sum of the lengths of the bases.

$$MN = \frac{1}{2}(QR + ZX)$$

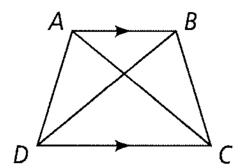


An *isosceles trapezoid* is a trapezoid in which the legs are congruent. An isosceles trapezoid has some special properties:

Each pair of base angles is congruent.



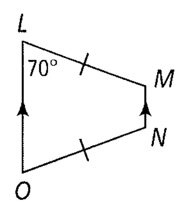
The diagonals are congruent.



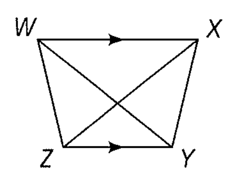
$AC = BD$

Exercises

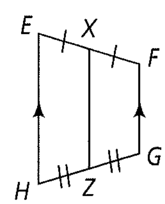
1. In trapezoid *LMNO,* what is the measure of $\angle OLM$? 70
What is the measure of $\angle LMN$? 110



2. *WXYZ* is an isosceles trapezoid and $WY = 12$. What is *XZ*? 12



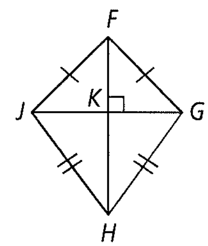
3. $\overline{XZ}$ is the midsegment of trapezoid *EFGH.* If $FG = 8$ and $EH = 12$, what is *XZ*? 10



Name ______________________ Class ____________ Date ____________

Reteaching (continued)

Trapezoids and Kites

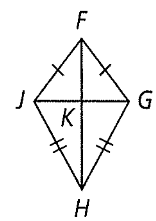


A *kite* is a quadrilateral in which two pairs of consecutive sides are congruent and no opposite sides are congruent.

In a kite, the diagonals are perpendicular. The diagonals look like the crossbars in the frame of a typical kite that you fly.

Notice that the sides of a kite are the hypotenuses of four right triangles whose legs are formed by the diagonals.

Problem



Write a two-column proof to identify three pairs of congruent triangles in kite *FGHJ*.

Statements	Reasons
1) $m\angle FKG = m\angle GKH = m\angle HKJ = m\angle JKF = 90$	1) Theorem 59
2) $\overline{FG} \cong \overline{FJ}$	2) Given
3) $\overline{FK} \cong \overline{FK}$	3) Reflexive Property of Congruence
4) $\triangle FKG \cong \triangle FKJ$	4) HL Theorem
5) $\overline{JK} \cong \overline{KG}$	5) CPCTC
6) $\overline{KH} \cong \overline{KH}$	6) Reflexive Property of Congruence
7) $\triangle JKH \cong \triangle GKH$	7) SAS Postulate
8) $\overline{JH} \cong \overline{GH}$	8) Given
9) $\overline{FH} \cong \overline{FH}$	9) Reflexive Property of Congruence
10) $\triangle FJH \cong \triangle FGH$	10) SSS Postulate

So $\triangle FKG \cong \triangle FKJ$, $\triangle JKH \cong \triangle GKH$, and $\triangle FJH \cong \triangle FGH$.

Exercises

In kite *FGHJ* in the problem, $m\angle JFK = 38$ and $m\angle KGH = 63$. Find the following angle and side measures.

4. $m\angle FKJ$ 90

5. $m\angle FJK$ 52

6. $m\angle FKG$ 90

7. $m\angle KFG$ 38

8. $m\angle FGK$ 52

9. $m\angle GKH$ 90

10. $m\angle KHG$ 27

11. $m\angle KJH$ 63

12. $m\angle JHK$ 27

13. If $FG = 4.25$, what is JF? 4.25

14. If $HG = 5$, what is JH? 5

15. If $JK = 8.5$, what is GJ? 17

Name ______________________ Class ____________ Date ____________

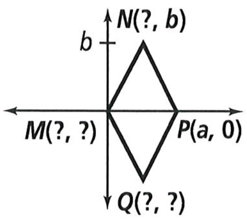
Reteaching

Applying Coordinate Geometry

You can use variables instead of integers to name the coordinates of a polygon in the coordinate plane.

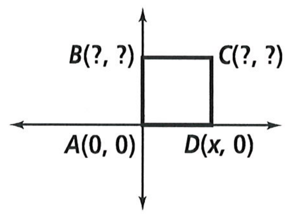
Problem

Use the properties of each figure to find the missing coordinates.



rhombus *MNPQ*

M is at the origin (0, 0). Because diagonals of a rhombus bisect each other, N has x-coordinate $\frac{a}{2}$. Because the x-axis is a horizontal line of symmetry for the rhombus, Q has coordinates $(\frac{a}{2}, -b)$.



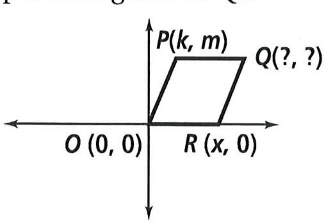
square *ABCD*

Because all sides are congruent, D has coordinate $(0, x)$. Because all angles are right, C has coordinates (x, x).

Exercises

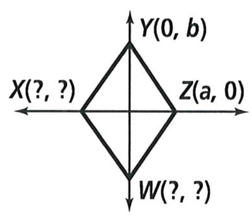
Use the properties of each figure to find the missing coordinates.

1. parallelogram *OPQR*



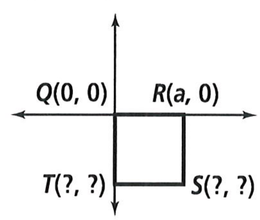
$Q(x + k, m)$

2. rhombus *XYZW*



$X(-a, 0)$; $W(0, -b)$

3. square *QRST*



$S(a, -a)$, $T(0, -a)$

4. A quadrilateral has vertices at $(a, 0)$, $(-a, 0)$, $(0, a)$, and $(0, -a)$. Show that it is a square.

Sample: Each side has a length of $a\sqrt{2}$, making the figure a rhombus. One pair of opposite sides has a slope of 1, and the other pair has a slope of -1. Because the product of the slopes is -1, the sides are perpendicular and the rhombus is a square.

5. A quadrilateral has vertices at $(a, 0)$, $(0, a + 1)$, $(-a, 0)$, and $(0, -a - 1)$. Show that it is a rhombus.

Each side has a length of $\sqrt{2a^2 + 2a + 1}$. Therefore, the figure is a rhombus.

6. Isosceles trapezoid *ABCD* has vertices $A(0, 0)$, $B(x, 0)$, and $D(k, m)$. Find the coordinates of C in terms of x, k, and m. Assume $\overline{AB} \parallel \overline{CD}$. $C(x - k, m)$

Name ______________________ Class __________ Date __________

Reteaching (continued)

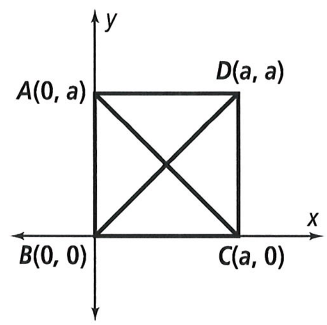
Applying Coordinate Geometry

You can use a *coordinate proof* to prove geometry theorems. You can use the Distance Formula, the Slope Formula, and the Midpoint Formula when writing coordinate proofs. With the Midpoint Formula, using multiples of two to name coordinates makes computation easier.

Problem

Plan a coordinate proof to show that the diagonals of a square are congruent.

Draw and label a square on a coordinate grid. In square *ABCD*, $AB = BC = CD = DA$. Draw in the diagonals, $\overline{AC}$ and $\overline{BD}$.



Prove that $AC = BD$. Use the Distance Formula.

$$CA = \sqrt{(0 - a)^2 + (a - 0)^2} = \sqrt{a^2 + a^2} = \sqrt{2a^2}$$

$$BD = \sqrt{(a - 0)^2 + (a - 0)^2} = \sqrt{a^2 + a^2} = \sqrt{2a^2}$$

So, $CA = BD$. The diagonals of the square are congruent.

Exercises

7. How would you use a coordinate proof to prove that the diagonals of a square are perpendicular?
   Answers may vary. Sample: Use the Slope Formula to prove that the product of the slopes of the diagonals is -1.
8. How would you use a coordinate proof to prove that the diagonals of a rectangle are congruent?
   Answers may vary. Sample: Use the Distance Formula to prove that the lengths of the diagonals are equal.
9. How would you use a coordinate proof to prove that if the midpoints of the sides of a trapezoid are connected they will form a parallelogram?
   Answers may vary. Sample: Use the Midpoint Formula to find the midpoints, then use the Slope Formula to show that opposite sides in the new figure have equal slopes.
10. How would you use a coordinate proof to prove that the diagonals of a parallelogram bisect one another?
    Answers may vary. Sample: Use the Midpoint Formula to show that the midpoints of the diagonals are the same point.
11. Classify quadrilateral *ABCD* with vertices $A(0, 0)$, $B(a, -b)$, $C(c, -b)$, $D(a + c, 0)$ as precisely as possible. Explain.
    isosceles trapezoid; one pair of sides parallel, other opposite pair of sides ≅, and diagonals ≅
12. Classify quadrilateral *FGHJ* with vertices $F(a, 0)$, $G(a, 2c)$, $H(b, 2c)$, and $J(b, c)$ as precisely as possible. Explain.
    Trapezoid; one pair of sides is parallel, the other pair of sides is not parallel.

Name ______________________ Class __________ Date __________

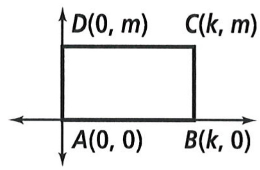
Reteaching

Proofs Using Coordinate Geometry

A *coordinate proof* can be used to prove geometric relationships. A coordinate proof uses variables to name coordinates of a figure on a coordinate plane.

Problem

Use coordinate geometry to prove that the diagonals of a rectangle are congruent.



$AC = \sqrt{(k-0)^2 + (m-0)^2}$

$= \sqrt{k^2 + m^2}$

$BD = \sqrt{(0-k)^2 + (m-0)^2}$

$= \sqrt{(-k)^2 + m^2}$

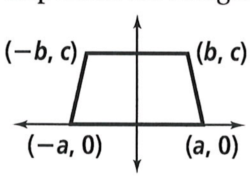
$= \sqrt{k^2 + m^2}$

$\overline{AC} \cong \overline{BD}$

Exercises

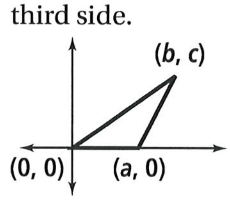
Use coordinate geometry to prove each statement.

1. Diagonals of an isosceles trapezoid are congruent.



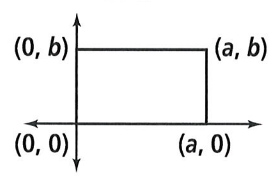
Use the Distance Formula to find the lengths of the diagonals. Each has a length of $\sqrt{c^2 + (b + a)^2}$. So, the diagonals are congruent.

2. The line containing the midpoints of two sides of a triangle is parallel to the third side.



You need to use the Midpoint Formula to find the midpoints of two sides, and the Slope Formula to show that the line connecting the midpoints is parallel to the third side. The midpoints are $(\frac{b}{2}, \frac{c}{2})$ and $(\frac{b+a}{2}, \frac{c}{2})$. The line connecting the midpoints has slope 0 and is therefore parallel to the third side.

3. The segments joining the midpoints of a rectangle form a rhombus.



You need to find the midpoints using the Midpoint Formula and lengths of segments connecting the midpoints with the Distance Formula. The midpoints are $(\frac{a}{2}, 0)$, $(a, \frac{b}{2})$, $(\frac{a}{2}, b)$, and $(0, \frac{b}{2})$. The segments joining these points are congruent, as they each have a length of $\frac{1}{2}\sqrt{a^2 + b^2}$.

Reteaching (continued)

Proofs Using Coordinate Geometry

The example used the Distance Formula to prove two line segments congruent. When planning a coordinate proof, write down the formulas that you will need to use, and write what you can prove using those formulas.

Exercises

State whether you can reach each conclusion below using coordinate methods. Give a reason for each answer.

4. $AB = \frac{1}{2}CD$.
   Yes; use the Distance Formula to show that $AB = \frac{1}{2}CD$.
5. $\triangle ABC$ is equilateral.
   Yes; use the Distance Formula to show all sides are equal.
6. Quadrilateral $ABCD$ is a square.
   Yes; use the Distance Formula to show all sides are equal and the Slope Formula to show all sides are perpendicular if the product of the slopes of any two adjacent sides is -1.
7. The diagonals of a quadrilateral form right angles.
   Yes; use the Slope Formula to show the diagonals are $\perp$ if the product of their slopes is -1.
8. Quadrilateral $ABCD$ is a trapezoid.
   Yes; use the Slope Formula to show that one pair of sides is parallel because the slopes are equal.
9. $\triangle ABC$ is a right triangle.
   Yes; use the Slope Formula to show that the product of the slopes of two sides is -1.
10. Quadrilateral $ABCD$ is a kite.
    Yes; use the Distance Formula to check that there are two pairs of adjacent sides of the same length, and that all four sides are not the same length.
11. The diagonals of a quadrilateral form angles that measure 30 and 150.
    No; I do not have coordinate methods to measure angles.
12. $m\angle D = 33$
    No; I do not have coordinate methods to measure angles.
13. $\triangle ABC$ is scalene.
    Yes; use the Distance Formula to show that all sides have a different length.
14. The segments joining midpoints of an equilateral triangle form an equilateral triangle.
    Yes; use the Midpoint Formula to find the midpoints of the sides and the Distance Formula to show that the lengths of all three sides are equal in the new triangle.
15. Quadrilateral $KLMN$ is an isosceles trapezoid.
    Yes; use the Slope Formula to show that one pair of sides is parallel (have equal slopes), and use the Distance Formula to show that the legs are congruent.